

A Method for Optimization of South Window Areas in Houses

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ABSTRACT

A simple graphical procedure is presented for calculating the heating energy requirement of direct-gain, passive solar houses during the design process in order to establish the combination of south window area and thermal storage mass that will minimize purchased space heating energy. It is applicable to the complete range of house construction types, from high-mass, direct-gain passive solar houses to low-mass, light-frame houses. An example illustrates the use of the method in establishing the optimum south-facing window area.

INTRODUCTION

The need for a simple method of estimating the heating energy requirement of a house is widely recognized. Such a technique could be used for comparing alternatives in the design process and for evaluating house designs to determine their compliance with energy standards. An important aspect of such a method is that it provides a means of estimating the amount of solar energy collected through windows and utilized to offset heat losses.

A number of methods have been developed for estimating the net solar contribution to space heating of direct-gain passive solar house designs. The solar-load ratio (SLR) method developed by the Los Alamos Scientific Laboratory^{1,2} is widely used in the United States and Canada, but the range of parameters that can be considered is limited to those in the reference designs. In particular, the SLR method cannot be used to examine the effects on a specific design of factors such as number of glazings and amount of thermal storage.

Another approach to estimating solar contribution is the "un-utilizability" method developed at the University of Wisconsin.³ It relies on solar radiation statistics to determine the non-useful fraction of solar gain that must be eliminated to prevent overheating. More parameters can be considered with it than with the SLR method, but it requires more calculations involving radiation data. It is, therefore, not so widely used as the SLR method. Recently, however, the "un-utilizability" method has been made easier to apply through the use of tables and graphs.⁴

The procedure presented in this paper combines the simplicity of the SLR method with the flexibility of the "un-utilizability" method. It permits calculation of the heating energy requirements of conventional as well as direct-gain passive solar houses and is applicable to the complete range of housing types, from high-mass, direct-gain to low-mass, sun-tempered houses. The calculation can be performed on a seasonal rather than on a monthly basis, making it easier to use and permitting its incorporation in a graphical technique that can be applied to determine the optimum* combination of south-facing window area and thermal storage mass. The graphical technique can also be used to illustrate the sensitivity of seasonal heating energy requirements to changes in glass area, type of glazing, and thermal mass. Examples for a Canadian location (Ottawa) indicate that this sensitivity is less than might be expected.

* Optimum in the sense of minimum heating energy (not an economic optimum).

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The energy consumption values measured in the NRCC passive test units during the 1981-82 heating season have been compared with corresponding values calculated by means of the method presented in this paper.⁵ Calculated values were in close agreement with the measured results, the maximum difference amounting to less than 5%.

UTILIZATION FACTOR

The calculation procedure is based on the concept of the solar utilization factor, η_s , defined as the fraction of total solar gain through all windows of a house that contributes to reduction of the heating requirement; that is,

$$\eta_s = \frac{\text{Useful solar gain}}{\text{Total solar gain}}$$

The useful solar gain for any hour includes the solar gain used to offset heat losses during that hour plus the portion stored in the thermal mass that is used to offset losses at a later time. It does not include excess gain that must be discarded to prevent room temperature from exceeding a preset maximum or any gain utilized to offset additional losses caused by a rise in room temperature above the thermostat setting.

The seasonal heating energy requirement, H, for a house is given by:

$$H = L - \eta_s G_s \quad (1)$$

where

- L = net heating load
- η_s = utilization factor for solar gain
- G_s = total solar heat gain through windows

The solar utilization factor is expressed as a function of two normalized parameters, the "gain-load ratio" (GLR) and the "thermal mass-gain ratio" (MGR). The gain-load ratio is the ratio of solar gain through windows, G_s , to the net heating load, L:

$$\text{GLR} = \frac{G_s}{L} \quad (2)$$

This gain-load ratio differs from the solar load ratio, as defined by Balcomb et al.,⁶ in that the GLR includes heat losses through windows while the SLR does not. Moreover, the GLR includes solar gains through all windows, not just those facing south.

The mass-gain ratio reflects the thermal storage characteristics of the building as well as the area, type, and orientation of the glazing. It is defined as

$$\text{MGR} = C/g_s \quad (3)$$

where

- C = thermal capacity of the building interior, MJ/K
- g_s = average hourly solar gain for season, MJ/hr,
($g_s = G_s/h$ in heating season)

The thermal capacity, C, is calculated as the "effective mass" of the building multiplied by its specific heat. The effective mass is the mass actually available for storing heat from direct solar gain or from close contact with room air so that any change in room air temperature affects the mass temperature. This normally includes the mass inside the insulating layer of the walls and ceilings, but excludes the exposed concrete of uninsulated basement walls and floors. Typical thermal capacity values for four types of construction are given in Tab. 1.

Solar utilization factors were derived by Barakat and Sander using hour-by-hour computer simulation of a large number of houses having different combinations of space heating load, thermal storage mass, solar gain, and allowable temperature swing for five Canadian locations.⁷ When the resulting utilization factors were correlated, using the parameters GLR and MGR, they were found to be independent of geographical location (Fig. 1). Figure 1 shows the solar utilization factor plotted against the GLR for various values of MGR and a room temperature rise of 5.5 C deg. Curves for 0 and 2.75 C deg temperature rise are also available.⁸

CALCULATION OF HEATING REQUIREMENT

The basis of the calculation method is the simple heat balance equation of the house, as given by Eq 1. The seasonal net heating load, L, is calculated as

$$L = L_t + L_a + L_b - \eta_i G_i \quad (4)$$

where

L_t = total heat loss due to transmission through exterior walls, windows, ceilings, etc.
 L_a = total heat loss due to indoor-outdoor air exchange (infiltration + ventilation)
 L_b = total below-grade heat loss
 η_i = utilization factor for internal gains
 G_i = total heat gains from internal sources (lights, equipment, people, etc.)

The seasonal total heat losses due to transmission through the exterior walls, windows, and ceiling may be calculated as

$$L_t = 0.036N(\bar{\theta}_i - \bar{\theta}_o)\sum_{(\text{components})} UA \quad (\text{MJ}) \quad (5)$$

where

N = number of hours in heating season
 $\bar{\theta}_i$ = seasonal average indoor air temperature
 $\bar{\theta}_o$ = seasonal average outdoor air temperature
UA = thermal conductance of each component, including all windows

The conductance values, UA, can be calculated as described in the ASHRAE Handbook,⁹ and $\bar{\theta}_o$ is available for many locations from weather data records.¹⁰ The heat losses due to indoor-outdoor air exchange, L_a , can be calculated from the equation,

$$L_a = 0.3353(\text{ACH})VN(\bar{\theta}_i - \bar{\theta}_o) \quad (\text{MJ}) \quad (6)$$

where

ACH = average air change rate of house (air changes per hour)
V = volume of house, m³

The total below-grade loss can be calculated using a procedure recently developed by Mitalas.¹¹ The magnitude of the internal gains, G_i , depends on the occupancy of the house. While data concerning the rate of heat release from various appliances are available, it is always necessary to make arbitrary assumptions regarding occupant behavior and, therefore, of internal gains. For most situations in which G_i is less than 25% of the heat loss, all internal gains are assumed to be useful ($\eta_i = 1$).

The seasonal total solar gain through the windows is calculated as

$$G_s = \sum_i A_i SC_i \phi_i \quad (7)$$

where

subscript i represents orientation of window, and
 A_i = glass area
 SC_i = shading coefficient of window
 ϕ_i = total solar gain through unit area of a single sheet of standard glass, given by
 $\phi_i = 0.83 H_i$, where H_i is the total seasonal solar radiation incident on the window surface

Values of total monthly solar radiation incident on surfaces of different orientations and tilts are tabulated for a number of locations. In Canada this information is available from the Atmospheric Environment Service as 10-year averages for 130 locations.⁷ The seasonal solar gain can be obtained by summing the values for the appropriate months.

Equations 2 and 3 are used to calculate the GLR and MGR. The utilization factor, η_s , can then be found from Fig. 1 and the space heating requirement (auxiliary heating) calculated from Eq 1.

GRAPHICAL DESIGN METHOD

Solar utilization may also be presented as the fraction of net heating load that must be supplied by the heating system. This fraction, F_h , will be referred to as the "purchased heating fraction." (It is, in fact, equal to one minus the solar heating fraction.)

Figure 2 shows F_h plotted as a function of the parameters GLR and MGR. The curves apply for a maximum allowable temperature swing of 5.5 C deg; graphs for other temperature swings have been given by Barakat and Sander.¹²

The heating energy requirement for the house can be obtained from

$$H = F_h L \quad (8)$$

The above procedure can be incorporated in a simple graphical method to examine the effect of south-facing window type and area as well as thermal storage on the heating requirement of a house. Figure 2 can be reformatted as shown in Fig. 3 and the following plotted against south window area for a particular house design:

1. Solar gain, G_s , calculated as a linear function of south window area, assuming that the areas of windows on the other orientations remain fixed:

$$G_s = G_o + (SC \phi A)_{\text{south}} \quad (9)$$

where

G_o is the solar gain through all orientations other than south

2. Net house load, L , calculated as a linear function of south window area:

$$L = (L'_t + L_a + L_b - \eta_i G_i) + 0.036(U_{\text{window}} - U_{\text{wall}})(\bar{\theta}_i - \bar{\theta}_o)NA_{\text{south}} \quad (10)$$

where

L'_t is the transmission loss of the house with all south windows replaced by wall ($A_{\text{south}} = 0$)

3. GLR, obtained by dividing G_s by L at a number of points
4. MGR, obtained by computing CN/G_s for a number of values of A_{south}

The procedure for constructing the relation between window area and purchased heating requirement is as follows (Fig. 3):

1. For any south window area, draw a vertical line to intersect the solar gain line, the net load line, the GLR curve, and the MGR curve at points A, B, C, and D, respectively.
2. Draw a horizontal line from point C to intersect the purchased heating fraction plot for the appropriate MGR (measured at D) at point E (some interpolation between MGR curves may be necessary). This determines the purchased heating fraction, F_h . The heating requirement, H , is then equal to the product of F_h and L . This is obtained graphically in the next three steps.
3. Draw a horizontal line from point B to intersect the vertical line of $F_h = 1.0$ at X. Connect a line between point X and point Y (point of intersection of the vertical line through $F_h = 0$ and the zero energy line).
4. Draw a vertical line from point E to meet the line XY at F.
5. Draw a horizontal line from point F to meet the original vertical line of Step 1 at point G. The energy value at point G represents the purchased heating energy associated with this south window area.
6. Repeat steps 1 through 5 for other south window areas and draw a curve through all G points. This curve represents the relation between A_{south} and purchased heating energy.

A change in thermal storage for the same house can be accommodated simply by calculating new values of MGR and repeating the above. A change in window type requires repetition of the entire procedure, beginning by recalculation of both G_s and L.

SAMPLE CALCULATIONS

Examples are presented in Figs. 4 and 5 for two different house envelope constructions (thermal resistance) in Ottawa, Canada. For each example, three options were evaluated:

1. light frame construction with all windows double glazed
2. same as 1, but with four times the amount of thermal storage
3. light construction with all windows triple glazed.

Details of the six cases examined are given in Tab. 2 along with the minimum purchased heating required for each case and the optimum south window area.

SUMMARY AND OBSERVATIONS

A simple graphical procedure is presented for determining the optimum combination of south window area, window type, and thermal storage mass for any house construction. The following observations regarding direct-gain passive solar houses in the Canadian climate were made by applying the method to a number of house designs for the Ottawa area.

The curves of purchased heating versus south-facing window area are very shallow around the optimum area. In general, a 50% change in south window area on either side of the optimum value results in a small change in purchased energy requirement (a maximum of 4% in the six cases presented).

The optimum south window area decreases for the more energy conserving houses. For double-glazed windows and lightweight construction the optimum area is about 8% of floor area for an 80 GJ house (House 1). This reduces to 3.5% of floor area for the more energy conserving (48 GJ) house (House 2).

Although an increase in thermal storage allows use of more windows and results in a reduction in purchased energy, a larger reduction in purchased energy can be achieved with a smaller area of triple-glazed windows. Taking the optimum double-glazing cases as bases for comparison, a four-fold increase in mass results in a 7% reduction in purchased heating for House 1 and 2% reduction for House 2. Use of triple-glazing results in reductions of 15% and 21% for Houses 1 and 2, respectively.

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3. W.A. Monsen, S.A. Klein, and W.A. Beckman, "Predicting of Direct Gain Solar Heating System Performance," Solar Energy, 27:2 (1981) p. 143-147.
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5. S.A. Barakat, and D.M. Sander, Utilization of Solar Gain through Windows for Heating Houses, National Research Council Canada, Division of Building Research, BRNote 184, 1982.
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9. ASHRAE Handbook -- 1981 Fundamentals Volume, Chapter 23.
10. Canada, Atmospheric Environment Service, Downsview, Ontario, Canadian Normals - Temperature, 1975.

11. G.P. Mitalas, Basement Heat Loss Studies at DBR/NRC, National Research Council Canada, Ottawa, Division of Building Research, NRCC 20416, (1982), 63 p.

12. Barakat and Sander.

TABLE 1
Sample House Weights

Thermal Capacity per Floor Area (MJ/m ² K)	Construction
0.060	<u>Light</u> - Standard frame construction, 12.7-mm gypsum board finish on walls and ceilings, carpet over wooden floor
0.153	<u>Medium</u> - As above, but 50.8-mm gypsum board finish on walls and 25.4-mm on ceiling
0.415	<u>Heavy</u> - Interior wall finish of 101.6-mm brick, 12.7-mm gypsum board finish on ceiling, carpet over wooden floor
0.810	<u>Very Heavy</u> - Commercial office building, 304.8-mm concrete floor

TABLE 2
Sample Calculation for Houses in Ottawa

Parameter	House 1			House 2		
	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3
Floor area, m ²	207	207	207	207	207	207
Thermal capacity, MJ/K	12.4	49.6	12.4	12.4	49.6	12.4
Wall thermal resistance, m ² ·K/W	3.5	3.5	3.5	6.4	6.4	6.4
Ceiling thermal resistance, m ² ·K/W	5.2	5.2	5.2	10.5	10.5	10.5
Glazing type	Double	Double	Triple	Double	Double	Triple
South solar gain, MJ/m ²	1814	1814	1670	1814	1814	1670
Basement heat loss, GJ	20	20	20	17	17	17
Internal gains, GJ	15	15	15	15	15	15
Air change rate, 1/h	0.25	0.25	0.25	0.1*	0.1	0.1
Heat loss coefficient (at A _{South} = 0), m ² ·K/W	206	206	197	120	120	116
Optimum south window area, m ²	16	32	24	7	18	10
Minimum purchased heating, GJ	81	75.5	69	48	47	38

* Equivalent to 0.5 air changes/hour using air-to-air heat exchanger of 0.8 effectiveness.

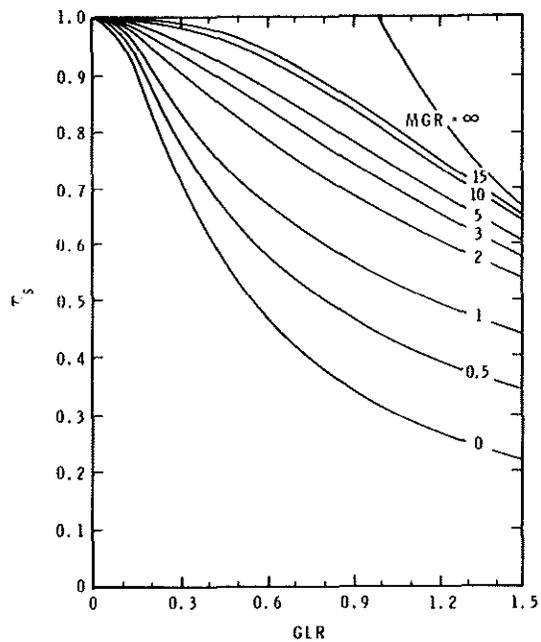


Figure 1. Seasonal solar utilization factor (room temperature swing = 5.5°C)

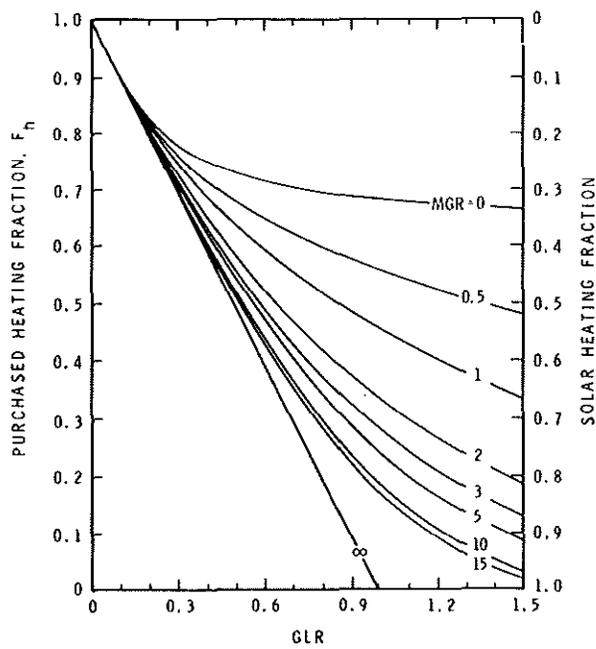


Figure 2. Seasonal purchased heating fraction (room temperature swing = 5.5°C)

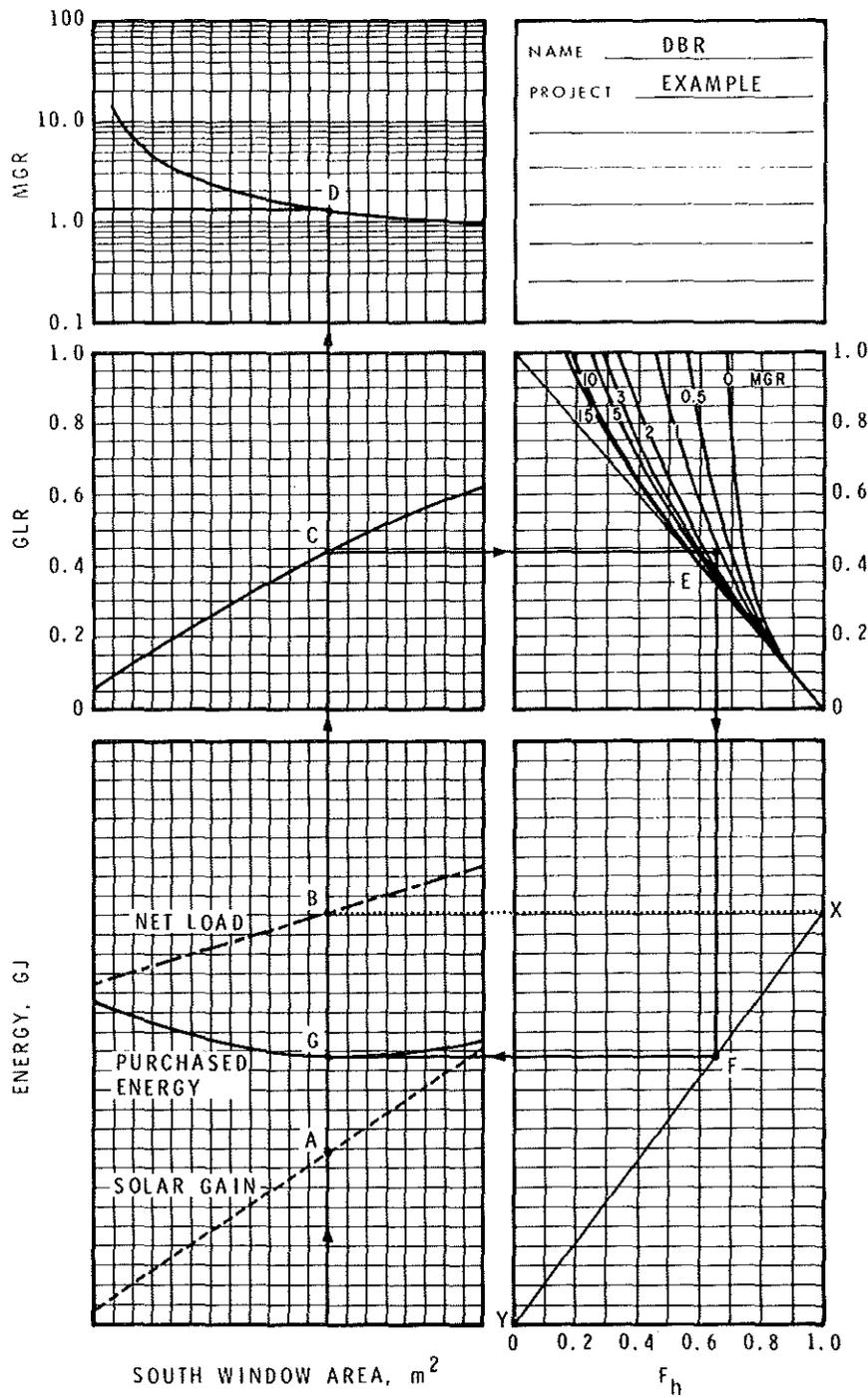


Figure 3. Illustration of graphical procedure

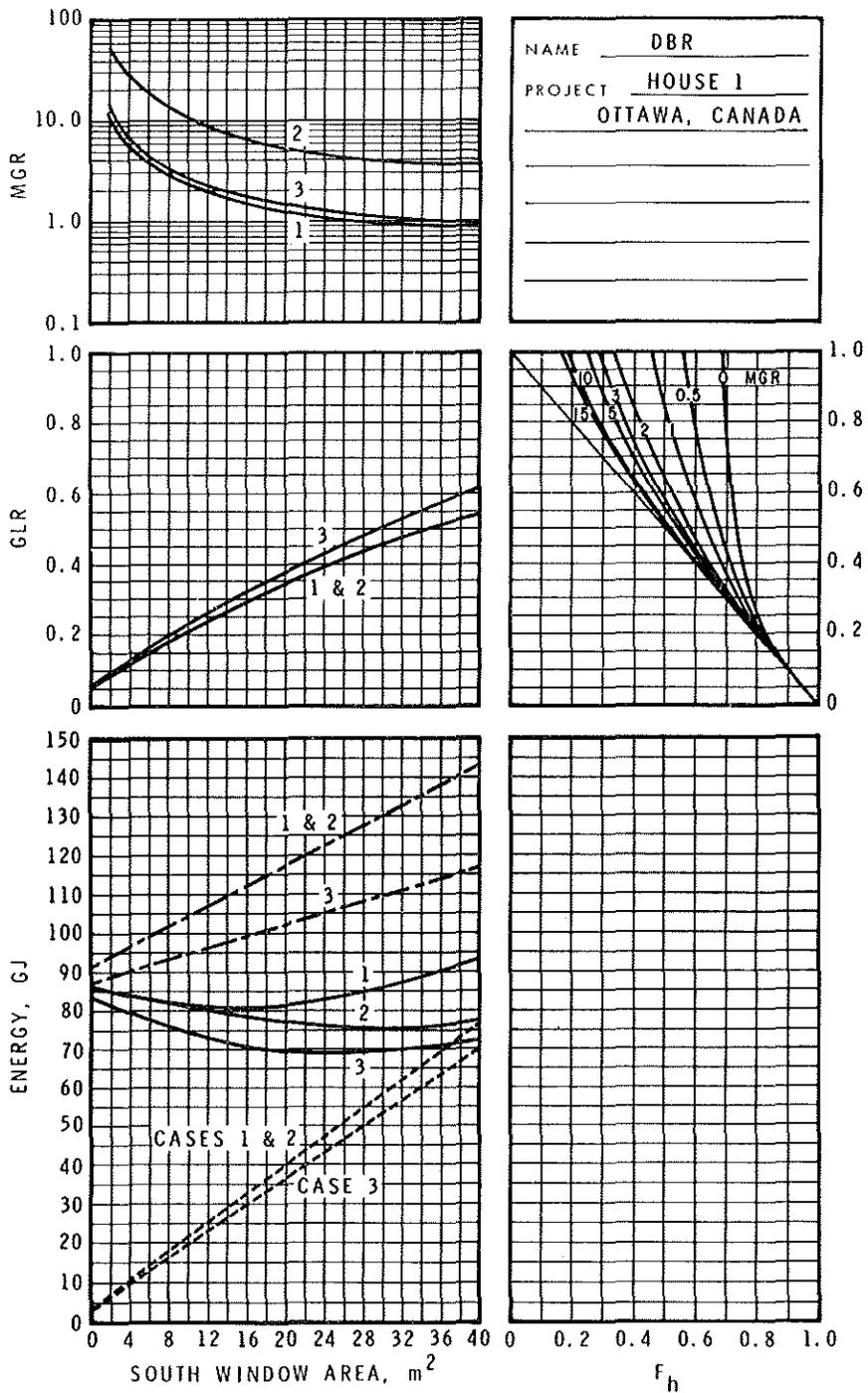


Figure 4. Sample calculation for House 1

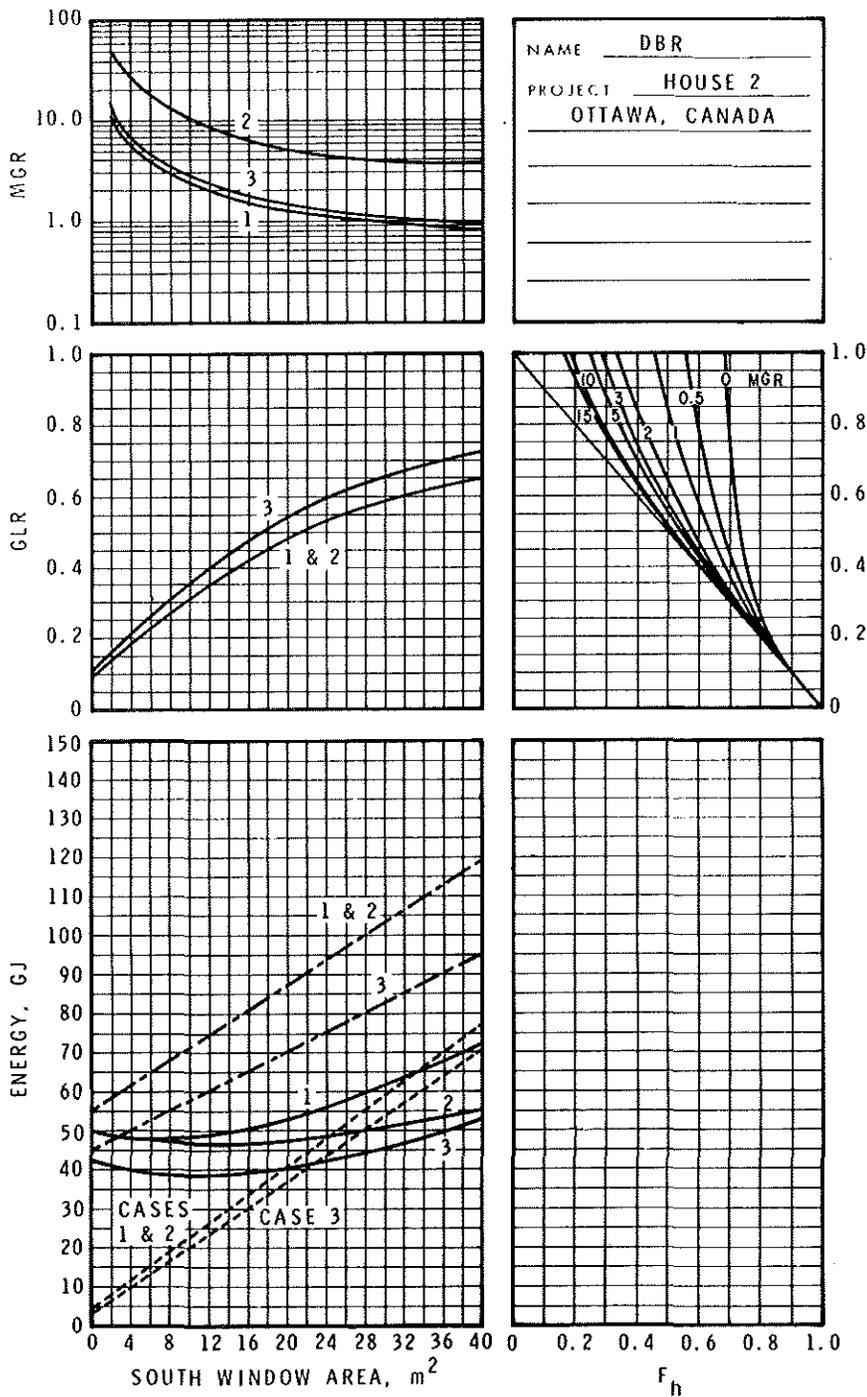


Figure 5. Sample calculation for House 2

Discussion

P. Brunello, University of Udine (Italy) - Lawrence Berkeley Lab., Berkeley, CA: Two questions about your interesting methodology for calculating requirements, whose philosophy seems to be very similar to a correlation procedure we have developed (reported in the Proceedings of the first ASHRAE-DOE conference held in Orlando, FL, in 1979). First, how do you take into account the variations of the internal radiation-convection laminar coefficient? Second, even though there is no internal air temperature swing, some solar energy impinging on the internal surfaces is lost, because it must be absorbed by the walls, raising their temperature and so increasing the conductive heat losses. Do you account for this phenomenon?

S.S. BARAKAT: The variations of the internal film coefficient are not accounted for in the calculations. These variations result from the change in wall surface temperature due to the incident solar radiation on part of the wall. No correlation for such variations exists at this time.

Since the response factors for solar gain are normalized, that is, all the gain is assumed to stay in the building, the excess heat loss due to rise in wall temperature is not taken into account in the computer calculations. However, this can be accounted for using the "F" factor described in the ASHRAE Handbook --1977 Fundamentals Volume, chapter 25, page 23. The useful solar gain into the building can be reduced by a factor less than 1.0 to account for these extra losses.

For well-insulated houses, such as those being built in Canada, the excess heat loss will be insignificant and the error due to their omission is much less than other errors associated with simple calculative methods, for example, due to uncertainty in solar radiation measurements or shortwave radiation reflected back through the window.

